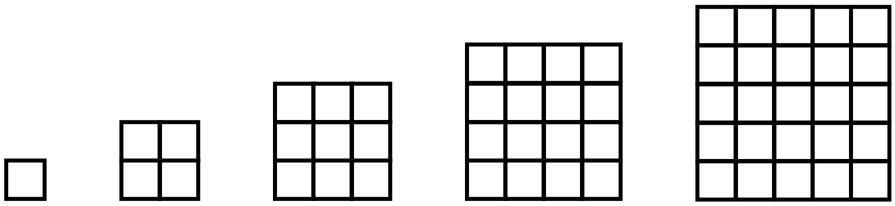
Patterning Math Lab 3

Opening Acts

1) Consider the following series of patterns of white boxes. Assume the series of patterns continues.

a) How many white boxes would be in the 10th pattern in the series?

b) What number pattern in the series would have 144 boxes?



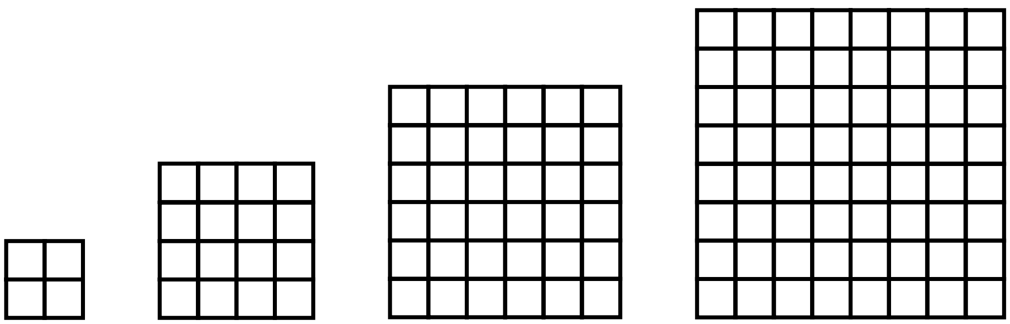
1 2 3 4 5

2) Consider the following series of patterns of white boxes. Assume the series of patterns continues.

a) How many white boxes would be in the 10th pattern in the series?

b) What number pattern in the series would have 256 boxes?

c) Would any pattern in the series have 121 boxes?



1 2 3 4

Main Event

3) Consider the following series of patterns of white and grey boxes. Assume the series of patterns continues.

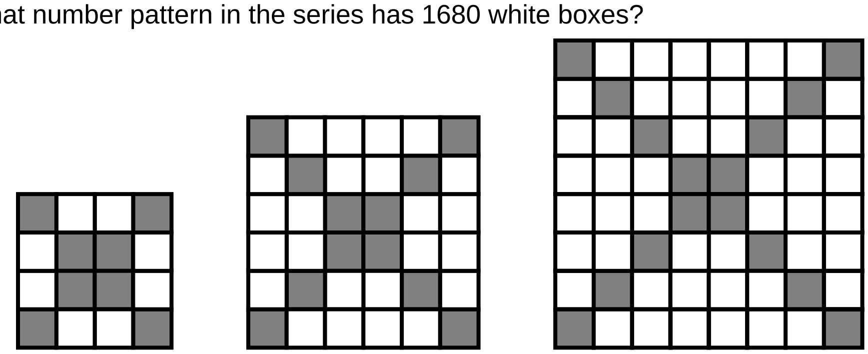
a) How many total boxes would be in the 9th pattern in the series?

b) How many grey boxes would be in the 12th pattern in the series?

c) What number pattern in the series has 100 grey boxes?

d) How many white boxes would be in the 15th pattern in the series?

e) What number pattern in the series has 1680 white boxes?



1 2 3

4) Consider question 1 from the Opening Act.

pattern number	# white boxes	first difference	second difference
1			
2			
3			
4			
5			

a) Copy the table into your lab notebook. Fill it out.
b) Do you notice a pattern in the # white boxes? Do you notice a pattern in the first differences? Do you notice a pattern in the second differences?
c) In Desmos, make a table with pattern number and # white boxes, along with the associated plot.
d) Should you connect the points with line segments?
e) You should see that the points are only plotted in one region of the displayed graph. Adjust the graph window as shown in Window settings (search the Knowledge Base); set –0.1 < x < 10 and –0.1 < y < 30. Get in the habit of choosing an appropriate graph window.
f) Make a table and graph of pattern number and first difference.
g) Make a table and graph of pattern number and second difference.

5) Consider an object that moves in a straight line with constant acceleration 2 m/s^2.

t (s)			
0			
1			
2			
3			
4			
5			

a) Copy the table into your lab notebook.In the table, fill in the second column, which is for acceleration. Make sure to label the column, along with units. The third column will be for velocity. The fourth column will be for position.
b) The object is initially at rest. Use that information to fill in the appropriate entry in the table. Fill in the rest of the entries in the velocity column.
c) The object is initially at the origin. Use that information to fill in the appropriate entry in the table. Fill in the rest of the entries in the position column. You can do this using the graphical method you learned last week or with the algebraic method from this week.
d) Make a time and acceleration table in Desmos, along with the associated plot. What toolkit function is this?
e) Repeat, with the time and velocity information. What toolkit function is this?
f) Repeat, with the time and position information. What toolkit function is this?

From the physics reading this week, we learned that for ***constant acceleration***, there are a set of special case kinematics equations that can be used to determine the position and velocity of an object undergoing constant acceleration at any particular time.

Take, for example, $v = v_0 + at$, where $v = v(t)$ is the velocity as a function of time, the parameter a is the (constant) acceleration, and the parameter v_0 is the initial velocity (the velocity when $t = 0$).

We can re-write this as $v = at + v_0$; writing it in this way makes it clear that v is a **linear function** of t by comparison to the standard slope-intercept form of a line $y = mx + b$: $\Rightarrow \left\{ \begin{matrix} v = a\ t + v_0 \\ \Updownarrow \quad \Updownarrow \Updownarrow \quad \Updownarrow \\ y = mx + b \end{matrix} \right\}$ so that the slope of a velocity vs. time graph is the acceleration, and that the y-intercept is the initial velocity.

Next, consider the equation for position as a function of time for constant acceleration: $x = x_0 + v_0 t + \frac{1}{2}at^2$, where $x = x(t)$ is the position as a function of time, the parameter a is the (constant) acceleration, the parameter v_0 is the initial velocity, and the parameter x_0 is the initial position (the position when $t = 0$). This is a **quadratic function**: a function that depends on the input variable squared (raised to the second power), as opposed to a linear function which depends on the first power of the input variable. In the case of kinematics, we see in this formula the $\frac{1}{2}at^2$ term; it is the 2 in t^2 that indicates we have a quadratic function (and that there are no larger exponents in the function).

Re-writing as $x = \frac{1}{2}at^2 + v_0 t + x_0$ allows comparison to the **standard form of a quadratic function** $y = Ax^2 + Bx + C$ (as you will see in next week's reading, your precalculus book uses the parameters a, b, c instead of A, B, C but that is potentially confusing in this particular context since we use a for acceleration): $\Rightarrow \left\{ \begin{matrix} x = \tfrac{1}{2}a\,t^2 + v_0\,t + x_0 \\ \Uparrow \quad \Uparrow \quad \Uparrow \quad \Uparrow \quad \Uparrow \quad \Uparrow \\ \Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow \\ y = A\,x^2 + B\,x + C \end{matrix} \right\}$

6) Consider a quadratic function in the standard form.

a) In Desmos, type in $y = Ax^2 + Bx + C$, and add all sliders for A, B, and C. Set A = 1, B = 0, and C = 0; this gives $y = x^2$, the prototypical quadratic function (also called a parabola).
b) Leaving A = 1 and B = 0, change only C using the slider. What effect does changing C have on the graph of the function?
c) Set B = 0 and C = 0. Change only A using the slider. What effect does changing A have on the graph of the function? Pay particular attention to what happens when A is small vs. large, and when A is positive vs. negative.
d) Set A = 1 and C = 0. Change only B using the slider. What effect does changing B have on the graph of the function?
e) Pick some non-zero value for A and for C (they can be different values). Adjust B so that the graph intersects the y-axis at one point and the x-axis at two points. Click on the graph. If you look carefully, you should see 4 grey dots on the graph. One dot should intersect at the y-axis. Two dots should intersect at the x-axis. The fourth dot should be at the very bottom (if the graph opens upward) or the very top (if the graph opens downward). The dot that is at the very bottom or very top of the parabola indicates a point called the **vertex**. If you hover over a dot, you will see its coordinate. If you click and drag on the graph, you can read off any coordinate of interest.

7) Just as there are several forms for a linear function (e.g. the **slope-intercept form** $y = mx + b$ and the **point-slope form** $y - y_0 = m(x - x_0)$, there are several forms for a quadratic function. You have explored the **standard form of a quadratic function**. Next, you will explore the **vertex form** (also called the **transformation form**): $y = A(x-h)^2 + k$, where A, h, and k are parameters.

a) Enter $y = A(x-h)^2 + k$ in Desmos, and add all sliders for A, h, and k.
b) Click on the graph and note the coordinates of the vertex. Does anything happen to the vertex if you change just A? Return A to 1.
c) Now, change just h to some other value. Note the coordinates of the vertex. Try several values of h (both positive and negative); each time you change h, note the coordinates of the vertex.
d) Repeat, but this time change just k to several different values (both positive and negative), noting the coordinates of the vertex each time.
e) If you were to set h = 3 and k = –5, where would the vertex be? Test your prediction.

8) Solving (reverse evaluating) a quadratic function can be challenging; we'll spend time in the next week developing our skills to do this. One method is to use graphing. Let's say that we have the quadratic function $f(x) = 5x^2 + 3x - 2$. If we wanted to know what y equals when x = 6, we substitute and evaluate:
$f(6) = 5(6)^2 + 3(6) - 2 = 5(6 \cdot 6) + 3(6) - 2 = 5(36) + 3(6) - 2 = 180 + 18 - 2 = 196$. However, if we want to know what value of x gives $f(x) = 6$, then we need to solve the algebraic equation $6 = 5x^2 + 3x - 2 \Leftrightarrow 5x^2 + 3x - 2 = 6$. This is challenging – try it and see. Some of you might know some techniques, such as factoring (works great if the quadratic is easy to factor) or the quadratic formula (works all the time if the quadratic can be manipulated into the form $Ax^2 + Bx + C = 0$). For now, let's take advantage of Desmos.

a) Enter $y = 5x^2 + 3x - 2$ into Desmos. Enter $y = 6$ into Desmos. This gives the graph of this constant function. If the graphs of the two functions intersect, then that means we can find values of x such that $5x^2 + 3x - 2 = 6$. What are these values?
b) Find any values of x such that $-4.9x^2 + 20x + 5 = 10$.
c) Find any values of x such that $x^2 + 6x + 9 = 3$.
d) Find any values of x such that $3x^2 + 2x + 1 = 0$.

9) Return to the Opening Acts and the Main Event. We encourage you to think of multiple methods of approaching these puzzling patterns.