Patterning Math Lab 2a

In Math Lab 2a, you will investigate and reinforce material from the Week 2 pre-calculus reading and some related ideas. You will explore: Transformations of Linear Functions; Parallel and Perpendicular Lines.; Domain and Range restrictions; Intersections of lines; Difference tables - first and second differences; Power function graphs; Toolkit function graphs. Work on this until 10:45 at which point you should switch over to Math/Physics Lab 2b.

Part 0

- o If you haven't already, print out the Week 2physics Problem Set and the pre-calculus Problem Set, available at the program web-site blogs.evergreen.edu/patterning under Calendar: Week 2. Please get in the habit of printing these out no later than Monday when you are on campus. You should bring these printouts with you to Wednesday Workshop and of course to Thursday's Problem Session. You should attempt half of the Problem Sets by Wednesday Workshop and attempt all of the Problem Sets by Problem Session.
- o If you haven't already, log on to WAMAP and WebAssign and make sure to access both on-line Problem Set versions. Please get in the habit of accessing both on-line sets no later than Monday.
- o Make sure your Math Lab notebook is following similar guidelines as your Physics Lab notebook: name/contact information, Table of Contents which you are updating, space after each lab, each lab started on a new page, etc.

Transformations of Linear Functions (Ch 2.2 precalculus)

1. Make a graph of each x, $x + 1$, $x - 1$, $x + 2$, $x - 2$. Do you notice a pattern?
2. Predict what the graphs of $x + 3$, $x + 4$, $x - 3$, $x - 4$ will look like. Test your prediction. Compare your graphs with the graphs in Ch 2.2 p114. Hide all these graphs.
3. Make a graph of each of x, $2x$, $3x$, $1/2\ x$, $1/3\ x$, $-2x$, $-3x$. Note: to get ½ x, type ½, then the right arrow key or tab, then x. Otherwise, you will get $1/2x$ (what is the difference between ½ x and $1/2x$?) Do you notice a pattern in the graphs?
4. Predict what the graphs of $-1/2\ x$, $-1/3\ x$ will look like. Test your prediction. Compare your graphs with the graphs in Ch 2.2 p113.
5. Now use sliders in Desmos to repeat the transformations in Ch 2.2 p114, Example 3. First enter the equation f(x) = -3x + 0.5x given in the example. Next enter the expression y = mx + b and add sliders for both m and b. Set both m=1 and b=0 to get the line y=x. Now adjust the b and m sliders to make the two lines coincide.

Parallel and Perpendicular Lines

- Type in the equation f(x) = -3 + 0.5x from Example 3 Ch 2.2 p114. Add the parameterized equation y = mx + b with sliders for m and b again.
- What slider parameter do you use to make the two lines parallel (m or b)?
- What slider parameter can you freely adjust and keep the lines parallel?
- What slider parameter can you use to make the lines perpendicular?
- What is the value of the m slider for all lines perpendicular to the function f?
- What is the relationship between the slopes of perpendicular lines? Use the sliders to test your answer.

Domain and Range Restrictions

This part of the lab lets you experiment with domain and range restrictions in Desmos.

1. Find and watch the *How to: Restrict Domain and Range* video tutorial in the knowledge base under the wrench. Don't worry about the piecewise function definition part of the video.
2. Create the domain restricted line y = ½ x - 5 with x > 0 as the restriction. You'll enter y = ½ x - 5 {x > 0}
3. Now replace the 0 restriction in the previous expression with the variable k and make k a slider. Play with the k slider to see how to easily adjust the restrictions on the x input values of the function.
4. Watch the Tour on Restrictions under the question mark.
5. Now replace the restriction {y<4} in the second function (the quadratic $y = x^2 - 2$) with the restriction {y<k} where k is your slider. Play with the k slider to see how it affects the y output value of the function.

Intersections of Lines

Ch 2.2 p119 Example 13 shows how to find the intersection of two lines algebraically. This section of the lab shows how you can find the intersection graphically! You can use this approach to check your answer for any intersection problem that you have solved algebraically.

1. Type into Desmos the two equations from Ch 2.2 p119 Example 13.
2. Zoom in using Desmos to get the coordinates of the intersection point.
3. Does your Desmos answer match the answer obtained algebraically in the text?

Difference Tables

1. Enter the equation for the linear function f(x) = 3x - 5 into Desmos.
2. Click on the gear in the upper right of your Desmos list and then click on the table icon that says convert to table.
3. In your lab notebook, copy down the table from Desmos and make a third column. In the third column, record the differences between the numbers in the second column. This is called a *first difference* table. Be sure you take each entry and subtract the one below it. Note: you can put the difference of the top two numbers in the second column in the first entry in the third column. You'll end up with one fewer entries in the third column because there are one fewer differences than entries in the second column.
4. What do you notice about the first differences?
5. What do the first differences correspond to in the equation of the function? What do these correspond to in the graph of the function?
6. Make first difference tables for a few different linear functions, for example -2x + 3, ¼ x – 1, and two more of your choice. Does the relationship you noticed between the first differences and the equation and graph of the function from the previous step hold true?
7. Enter the equation for the quadratic function $g(x) = 3x^2 - 2x - 1$. We know this is a quadratic because of the x^2.
8. Click on the gear in the upper right of your Desmos list and then click on the table icon that says convert to table.
9. In your lab notebook, copy down the table from Desmos and make a third column of first differences. Now make a fourth column and record the differences of the third column. This third column is the differences of the first differences and these are called *second differences*. Again, be sure that when you do differences you take each entry and subtract the one below it.
10. What do you notice about the second differences?
11. Make second difference tables for a few different quadratic functions, for example $4x^2$, $-½ x^2 + 3$, and two more of your choice. For each function, what do you notice about the second differences?
12. How do the second differences appear to relate to the original quadratic?

Power Functions

Continue with the following exercise from Ch 2.2 p156. If necessary, complete as homework.

1. Graph the even power functions in Ch 2.2 p156. What is their long-run behavior?
2. Graph the odd power functions in Ch 2.2 p156. What is their long-run behavior?

Toolkit functions

Continue with the following exercise from Ch 1.1 p10. If necessary, complete as homework.

1. Graph all the Toolkit Functions in Ch 1.1 p10. We will add more Functions to the Toolkit later.
2. Check your graphs with the pictures in Ch 1.1 p11-12.

Patterning Math/Physics Lab 2b

In Math/Physics Lab 2b, you will use Video Analysis as you did in Physics Lab 2, but this time to investigate situations where the velocity is not constant. You may record your lab notes either in your Math Lab notebook or your Physics Lab notebook. Leave room in your notebook for adding in your printed out graphs; it makes sense to leave that space near where you are recording your responses to the corresponding questions. If you can't recall how to perform a task in LoggerPro, check your lab notes or references, then consult with a neighbor, then consult with a TA or instructor. In that event, make sure you have recorded the steps so you don't need to ask again.

You can find all the videos for today's lab in the program file share, under Handouts: Week 2 Lab. All the videos you will analyze today continue to be limited to motion in a straight line; some will have horizontal motion and some will have vertical motion. You will save your individual work to your own Cubbie.

Part 1. Slowing Down (source: J.A. Bryan, Ball State University)

a) Launch LoggerPro, and use Insert: Movie to insert the video Slowing Down. Watch the video a few times (straight through and using the scroll bar at the bottom of the video player screen).
b) Recall that a motion diagram is the simplified representation of the motion of the object produced by marking its position at a particular time with a dot. Produce a **motion diagram** for the video Slowing Down.
c) If the dots on the motion diagram were evenly spaced, what would that tell you about the motion of the object?
d) What do you notice about the actual spacing between the dots in the motion diagram for this video? What does this indicate about the motion of the object? (Hint: look at the title of the video again)
e) Produce a **position vs. time graph** (since the motion is in the x direction, this is an X vs. time graph).
f) Does the position vs. time graph appear linear to your eye? If it does, please check in with a neighbor, TA, or instructor. If it does not, do you recognize the shape (it's ok if you do not)?
g) Copy this graph to your document file for later printing; make sure to label it.
h) Produce a **velocity vs. time graph** (in this case, and X velocity vs. time graph).
i) Does the velocity vs. time graph appear linear to your eye? Fit a line to the velocity vs. time graph and obtain the slope of the line.
j) What physical quantity does the slope of a velocity vs. time graph represent? What does the sign represent?
k) Record the acceleration of the car. How does LoggerPro report the units? How does this match how your book reports units for acceleration?
l) Copy this graph to your document file for later printing; make sure to label it.
m) Save your usefully named LoggerPro file and your document file in your cubbie.

Part 2. Speeding Up (source: J.A. Bryan)

a) Note the non-standard frame rate for this video, indicated in the first frame. You can adjust for this using Movie Options (right click on the movie) and then Override frame rate to: .
b) Reproduce steps a) through l) as before for this video.

Part 3. Ball Toss (source: J.A. Bryan)

a) Produce a motion diagram. What occurs here that hasn't occurred in your previous motion diagrams?
b) Reproduce the rest of the analysis as previously, noting that this time the motion is in the Y direction.
c) As you are able, make connections (compare and contrast) your results and analysis of Slowing Down, Speeding Up, and Ball Toss.

Part 4. Up and Down, Up and Down (source: LivePhoto Physics Series, RIT)

a) Note the non-standard frame rate as well as the reference scale in the first frame.
b) Produce a motion diagram.
c) Produce a position vs. time graph. Does this look linear? Do you recognize this shape? (It's ok if you don't.)
d) Produce a velocity vs. time graph. Does this look linear? Do you recognize this shape? Compare it especially with the position vs. time graph's shape.
e) We can't do any further analysis right now. Save the LoggerPro file, etc.

Applications and Extensions

1. Consider your mathematical model for the One Buggy motion from Physics Lab 2. Use your model to answer the following questions.

a) Where will the buggy be at 0.5 seconds? Answer in terms of world coordinates, not in terms of the meter stick. This is equivalent to asking you to evaluate your linear function at $t = 0.5$ s.
b) When will the buggy be at $x = 0.5$ m (the position is in world coordinates, not in terms of the meter stick). This is equivalent to asking you to solve the linear function for $x = 0.5$ m.
c) Use your model to predict when the buggy will be at $x = 1.0$ m (world coordinates).
d) Test your Prediction using Test Your Prediction 1 in the Week 1 Lab folder in the program file share Handouts folder.

2. Perform video analysis on the video When Will They Pass (available in the Week 1 Lab folder) to answer the question: when will the buggies pass each other?

3. View the video Will They Collide (available in the Week 2 Lab folder). Get started in answering that question.