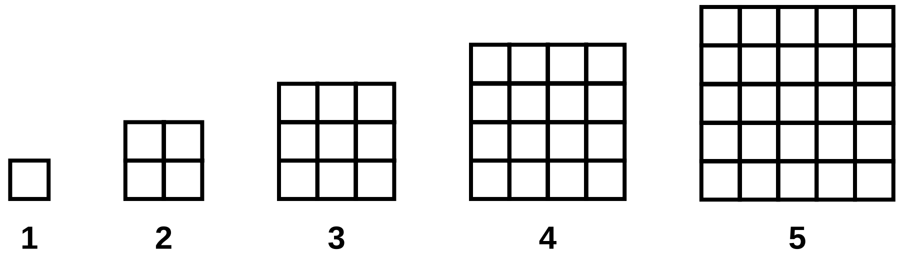
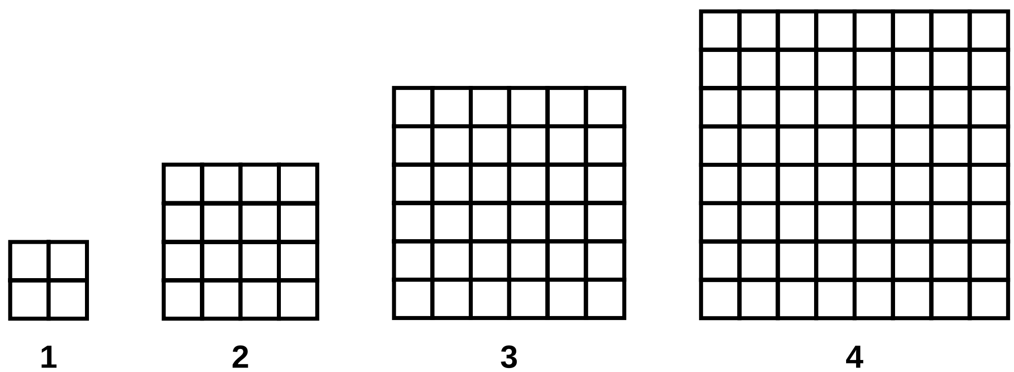
Patterning Math Lab 3

1) Consider the following series of patterns of white boxes. Assume the series of patterns continues.
a) How many white boxes would be in the 10^{th} pattern in the series?
b) What number pattern in the series would have 144 boxes?



2) Consider the following series of patterns of white boxes. Assume the series of patterns continues.
a) How many white boxes would be in the 10^{th} pattern in the series?
b) What number pattern in the series would have 256 boxes?
c) Would any pattern in the series have 121 boxes?



3) Consider question 1 above.

pattern number	# white boxes	first difference	second difference
1			
2			
3			
4			
5			

a) Copy the table into your lab notebook. Fill it out. See Math Lab 2 as needed to review how to calculate first differences and second differences.
b) Do you notice a pattern in the # white boxes? Do you notice a pattern in the first differences? Do you notice a pattern in the second differences?
c) In Desmos, make a table with pattern number and # white boxes, along with the associated plot of # white boxes vs. pattern number.
d) Should you show individual points or connect the points with line segments?
e) You should see that the points are only plotted in one region of the displayed graph. Adjust the graph window as shown in Window settings (search the Knowledge Base); set $-0.1 < x < 10$ and $-0.1 < y < 30$. Get in the habit of choosing an appropriate graph window.
f) Make a table of pattern number and first difference along with the associated plot of first difference vs. pattern number.
g) Make a table of pattern number and second difference and the associated plot of second difference vs. pattern number.
h) Hide all graphs.

4) Consider an object moving in a straight line with constant acceleration 2 m/s^2.

a) Copy the table into your lab notebook. In the table, fill in the acceleration column.
b) The object is initially at rest. Use that information to fill in the appropriate entry in the table. Fill in the rest of the entries in the velocity column using the fact that the acceleration is constant.
c) The object is initially at the origin. Use that information to fill in the appropriate entry in the table. Fill in the rest of the entries in the position column. You can do this using the graphical method you learned last week or with the algebraic method from this week. You might find it illuminating to try both approaches.
d) Make a time and acceleration table in Desmos, along with the associated plot.
e) Repeat, with the time and velocity information.
f) Repeat, with the time and position information.
g) Hide all graphs.

t (s)	x (m)	v (m/s)	a (m/s^2)
0			
1			
2			
3			
4			
5			

From the physics reading this week, we learned that for ***constant acceleration***, there are a set of special case kinematics equations that can be used to determine the position and velocity of an object undergoing constant acceleration at any particular time.

Take, for example, $v = v_0 + at$, where $v = v(t)$ is the velocity as a function of time, the parameter a is the (constant) acceleration, and the parameter v_0 is the initial velocity (the velocity when $t = 0$).
We can re-write this as $v = at + v_0$; writing it in this way makes it clear that v is a **linear function** of t by comparison to the standard slope-intercept form of a line $y = mx + b$: $\Rightarrow \left\{ \begin{matrix} v = & a & t & + v_0 \\ \Updownarrow & \Updownarrow & \Updownarrow & \Updownarrow \\ y = & m & x & + b \end{matrix} \right\}$ so that the slope of a velocity vs. time graph is the acceleration, and that the y-intercept is the initial velocity.

Next, consider the equation for position as a function of time for constant acceleration:
$x = x_0 + v_0 t + \frac{1}{2}at^2$, where $x = x(t)$ is the position as a function of time, the parameter a is the (constant) acceleration, the parameter v_0 is the initial velocity, and the parameter x_0 is the initial position (the position when $t = 0$). This is a **quadratic function**: a function that depends on the input variable squared (raised to the second power), as opposed to a linear function which depends on the first power of the input variable. In the case of kinematics, we see in this formula the $\frac{1}{2}at^2$ term; it is the 2 in t^2 that indicates we have a quadratic function (and that there are no larger exponents in the function).

Re-writing as $x = \frac{1}{2}at^2 + v_0 t + x_0$ allows comparison to the **standard form of a quadratic function** $y = Ax^2 + Bx + C$ (your precalculus book uses the parameters a, b, c instead of A, B, C but that is potentially confusing in this particular context since we use a for acceleration): $\Rightarrow \left\{ \begin{matrix} \underbrace{x} = & \underbrace{\tfrac{1}{2}a} & \underbrace{t^2} & + \underbrace{v_0} & \underbrace{t} & + \underbrace{x_0} \\ \Uparrow & \Uparrow & \Uparrow & \Uparrow & \Uparrow & \Uparrow \\ \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow \\ \overbrace{y} = & \overbrace{A} & \overbrace{x^2} & + \overbrace{B} & \overbrace{x} & + \overbrace{C} \end{matrix} \right\}$

5) Consider a quadratic function in the standard form.

a) In Desmos, enter $y = x^2$, the prototypical quadratic function (also called a parabola). Leave this there for comparison purposes.

b) In Desmos, enter $y = Ax^2 + Bx + C$, and add all sliders for A, B, and C. Set A = 1, B = 0, and C = 0; this should match $y = x^2$.

c) Leaving A = 1 and B = 0, change only C using the slider. What effect does changing C have on the graph of the function?

d) Set B = 0 and C = 0. Change only A using the slider. What effect does changing A have on the graph of the function? Pay particular attention to what happens when A is smaller than 1 vs. larger than 1, and when A is positive vs. negative.

e) Set A = 1 and C = 0. Change only B using the slider. What effect does changing B have on the graph of the function?

f) Pick some non-zero value for A and for C (they can be different values). Adjust B so that the graph intersects the y-axis at one point and the x-axis at two points. Click on the graph. If you look carefully, you should see up to 4 grey dots on the graph. One dot should intersect at the y-axis. Up to two dots should intersect at the x-axis. The fourth dot should be at the very bottom (if the graph opens upward) or the very top (if the graph opens downward). The dot that is at the very bottom or very top of the parabola indicates a point called the **vertex**. If you hover over a dot, you will see its coordinate. If you click and drag on the graph, you can read off any coordinate of interest.

g) Are you able to find a combination of A, B, and C such that the parabola does not have a y-intercept (make sure to zoom out as needed)? Why can't you find such a combination?

h) Are you able to find a combination of A, B, and C such that the parabola does not have a vertex (make sure to zoom out as needed)? Why can't you find such a combination?

i) Find some combination of A, B, and C such that the parabola has 2 x-intercepts.

j) Find some combination of A, B, and C such that the parabola has only 1 x-intercept.

k) Are you able to find a combination of A, B, and C such that the parabola has no x-intercepts?

l) In anticipation of the next part, delete the A slider. Hide all graphs.

6) Just as there are several forms for a linear function (e.g. the **slope-intercept form** $y = mx + b$ and the **point-slope form** $y - y_0 = m(x - x_0)$, there are several forms for a quadratic function. You have explored the **standard form of a quadratic function**. Next, you will explore the **vertex form** (also called the **transformation form**): $y = A(x-h)^2 + k$, where *A*, *h*, and *k* are parameters.

a) Enter $y = A(x-h)^2 + k$ in Desmos, and add all sliders for A, h, and k.

b) Click on the graph and note the coordinates of the vertex. Does anything happen to the vertex if you change just A? Return A to 1.

c) Now, change just h to some other value. Note the coordinates of the vertex. Try several values of h (both positive and negative); each time you change h, note the coordinates of the vertex.

d) Repeat, but this time change just k to several different values (both positive and negative), noting the coordinates of the vertex each time.

e) If you were to set h = 3 and k = –5, where would the vertex be? Test your prediction.

7) A quadratic function can be written either in standard form or in vertex form.

a) Review p. 166 in your pre-calculus text: Finding the Vertex of a Quadratic and Example 3.

b) Plot $y = 4x^2 - 12x - 1$ in Desmos and adjust the graph window so you can see the vertex, y-intercept, and any x-intercepts.

c) Using the formulas given on p. 166 and following as in Example 3, show that $y = 4x^2 - 12x - 1$ can be written in vertex form as $y = 4(x - \frac{3}{2})^2 + (-10)$.

d) Plot $y = 4(x - \frac{3}{2})^2 + (-10)$ and confirm that these two expressions have the same graph.

e) Confirm that the vertex you calculated is the same as the vertex shown on the graph.

8) Solving (reverse evaluating) a quadratic function can be challenging; we'll spend time developing our skills to do this. One method is to use graphing. Let's say that we have the quadratic function $f(x) = 5x^2 + 3x - 2$. If we wanted to know what y equals when x = 6, we substitute and evaluate:

$$f(6) = 5(6)^2 + 3(6) - 2 = 5(6 \cdot 6) + 3(6) - 2 = 5(36) + 3(6) - 2 = 180 + 18 - 2 = 196 .$$

However, if we want to know what value of x gives $f(x) = 6$, then we need to solve the algebraic equation:

$$6 = 5x^2 + 3x - 2 \Leftrightarrow 5x^2 + 3x - 2 = 6 .$$

This is challenging – try it and see. Some of you might know some techniques, such as factoring (works great if the quadratic is easy to factor) or the quadratic formula. For now, let's take advantage of Desmos.

a) Enter $y = 5x^2 + 3x - 2$ into Desmos. Enter $y = 6$ into Desmos, which gives the graph of this constant function. If the graphs of the two functions intersect, then that means we can find values of x such that $5x^2 + 3x - 2 = 6$. What are these values?
b) Let's try something similar for $5x^2 + 3x - 2 = -6$. Enter $y = 5x^2 + 3x - 2$ into Desmos. Enter $y = -6$ into Desmos. Do these graphs intersect? What does that mean about $5x^2 + 3x - 2 = -6$?
c) Find any values of x such that $-4.9x^2 + 20x + 5 = 10$.
d) Find any values of x such that $x^2 + 6x + 9 = 3$.
e) Find any values of x such that $3x^2 + 2x + 1 = 0$.

9) Solving quadratic equations turns out to be equivalent to finding the x-intercepts of a (related) function. The x-intercepts are also called the roots. These are values of x that when input into the function give an output of 0. Recall from your previous work that not all quadratics have real roots, some have 1 unique root, and some have 2 unique roots.

a) Let's return to $5x^2 + 3x - 2 = 6$. We can do some mathematics to get the equivalent equation $5x^2 + 3x - 8 = 0$. Show how you can turn $5x^2 + 3x - 2 = 6$ into $5x^2 + 3x - 8 = 0$.
b) Use Desmos to find any roots of $5x^2 + 3x - 8$ and confirm that these are the values for x that you got in part 8a).
c) Now, let's develop an analytical method to find roots (when they exist). Using the method of part 7c), show that you can re-write $5x^2 + 3x - 8$ into vertex form $5\left(x - (-0.3)\right)^2 + (-8.45)$, and confirm this graphically. It is perhaps simpler to write $5\left(x - (-0.3)\right)^2 + (-8.45)$ as $5\left(x + 0.3\right)^2 - 8.45$ which will do from now on.
d) Now, set $5\left(x + 0.3\right)^2 - 8.45$ equal to 0 to get $5\left(x + 0.3\right)^2 - 8.45 = 0$. Solve this for x. If you are uncertain of your algebra, work with a classmate or with an instructor or TA, making sure to go through each step as carefully as you need.
e) Use Desmos to confirm your answers.

10) Consider the following series of patterns of white and grey boxes. Assume the series of patterns continues.
a) How many total boxes would be in the 9th pattern in the series?
b) How many grey boxes would be in the 12th pattern in the series?
c) What number pattern in the series has 100 grey boxes?
d) How many white boxes would be in the 15th pattern in the series?
e) What number pattern in the series has 1680 white boxes?