Math Lab 8 - Periodic Functions

This math lab uses Desmos to support a close reading of Ch. 6 (and related sections) in the precalculus text.

Part 1 – Sinusoidal Graphs, Ch 6.1

1. In Desmos, input y = sin(theta). We'll use theta instead of x as the input variable to the sin function, though we could just as easily input y = sin(x).
2. Notice that the pattern repeats itself. At what approximate value of theta does the sin(theta) function repeat? This repeat is called the period P of the function.
3. Convert your horizontal axis labels in Desmos to multiples of π (via the Windows Settings wrench). Find an exact value for the period P of sin(theta).
4. Briefly describe in your own words how the full sine function can be obtained from the unit circle. You may find the picture on p.354 helpful.
5. Look at the sine graph. Propose a domain and range for your sin function. (The domain refers to the set of numbers that the input values can have and range refers to the set of numbers the output values can have.)
6. Write down how many angles you think are coterminal with $\pi/2$ radians (90 degrees). How does the graph of the sine function suggest an answer?
7. Input y = cos(theta). Compare and contrast with sin(theta).
8. Closely read p.355, the sections on Domain and Range of Sine and Cosine and Period of Sine and Cosine in light of your observations of the graphs of sin(theta) and cos(theta).

9. Compare the graphs of y = sin(theta), y = sin(-theta), y = -sin(theta), and y = -sin(-theta). Compare the graphs of y = cos(theta), y = cos(-theta), y = -cos(theta), and y = -cos(-theta). Summarize your observations. Hide the graphs.
10. Input y = sin(theta) and y = cos(h – theta), where h is a slider. Play with the slider, and note the transformational effect the h parameter has on the cosine function. For what minimum value of h does cos(h – theta) = sin(theta)? Give an exact value as a multiple of pi.
11. You were introduced to the *cofunction identity* sin(theta) = cos(90^{o} – theta) = cos($\pi/2$ – theta). See Ch 5.5 p345 in the precalculus text. Can you see the cofunction identity in your graphs of sin(theta) and cos(h – theta)?
12. Closely read the middle section on p.356 in light of your investigations of the symmetry of sin and cos and the effect of horizontal shifting. Hide the graphs.

13. Modify your sine function so it is y = A sin(theta) and make the A parameter a slider. Play with the A parameter and then describe in your lab notebook what transformational effect the A parameter has on the sine function. In your lab notebook do the “Try it Now” exercise on p.357. You can replace the cosine function with the sine function in the exercise. Use your A slider to verify your answer in Desmos rather than sketching the graph.

14. Modify your sine function so it is y = A sin(theta) + k and make the k parameter a slider. Play with the k parameter and then describe in your lab notebook what transformational effect the k parameter has on the sine function. In your lab notebook do the Try-it-Now exercise on p.358. Again, you can replace the cosine function with the sine function in the exercise. Use your k slider to verify your answer in Desmos rather than sketching the graph.

15. Modify your sine function so it is y = A sin(B t) and make the B parameter a slider. (Turn off your cosine function for now in Desmos). Play with the B parameter and then describe in your lab notebook what transformational effect the B parameter has on the sine function. Switch between the multiples-of-pi and regular numbers on the horizontal axis labels (via the wrench). Which labeling is more natural for the B parameter? Read closely the p.359 discussion on the second half of the page relating the B parameter to the

period P of the sine function. Experiment with the B slider to see what value it needs to make one complete period occur at t=2 and then t=1. Write those values down in your lab notebook and also propose a relationship between the period P and the parameter B. Don't just write down an equation to describe the relationship, rather, write down an explanation of the relationship in words that have meaning to you. Try entering a new function y = sin(2*pi/P t) with P a parameter. Experiment with the P parameter. Try the multiples-of-pi and the regular numbers for labeling the horizontal axis. Which labeling is more natural for the P parameter?

16. Read the top of p.361 and Example 7. Do the Try-it-Now at the bottom of p.361 using a formula in Desmos. Put the formula in your lab notebook and check your answer.

17. Read the bottom of p.363 giving the general formula for sine and cosine with all the transformation parameters. Read Example 10 (p.363) and Example 11 (p.364). Do the Try-it-Now exercise on p.365 and record in your lab notebook.

Part 2 - Graphs of the Other Trig Functions, Ch 6.2

Hide the graphs of the Desmos functions you have input so far. In this part of the lab, we mainly want you to see the graphs of the other trig functions. Start this part by reading Ch5.2 pp333-334 for the definitions of all the other trig functions. Notice how all the trig functions can be related back to the sine and cosine.

1. Look up the definition of the tangent function in Ch 5.4. Can you see how the tangent function is defined purely in terms of the sine and cosine functions? Write the definition of the tangent function in terms of the sine and cosine functions. Input the function y = tan(theta). What horizontal axis values are not defined? Can you explain why? Write down the domain and range of the tangent function using your own words. Also write down the succinct formulation of the domain and range given in the textbook. Hide the graphs.

2. What is the relationship between the secant function and the cosine function? Input the secant function y = sec(theta). Also input y = cos(theta). Read the top of p.371 top and make sure your Desmos graphs look like the graphs drawn. Hide the graphs.

3. What is the relationship between the cosecant function and the sine function? Input the cosecant function y = csc(t). Also input y = sin(t). Read the bottom of p.371 and make sure your Desmos graphs look like the graphs drawn.

4. Repeat for the cotangent function cot(theta) compared to tan(theta).

Part 3 - Solving Trig Equations, Ch 6.4

Recall that solving equations really means running functions in reverse, that is, finding all inputs that produce a given output. For example, given the positional function of a rocket over time, we would solve an equation to find the time at which the rocket reached a certain height. If the function is not one-to-one, then there may be multiple answers, or sometimes no answer. In the rocket example, there are sometimes multiple answers (the rocket passes a height going both up and down), there are heights above the rocket's highest point where there are no times to solve the equation. And there is one height where there is exactly one answer. In the case of the rocket, the position is given by a quadratic function and quadratics are not one to one on the full horizontal axis.

Similarly, recall that we can view solving equations as graph intersections. For example, we often solved quadratics by finding the horizontal intercepts, that is, the intersection of the horizontal axis and the quadratic. We can see the question of the time for a specific rocket height h (discussed above) as the intersection of the horizontal line y = h with the quadratic for the rocket position over time.

Ch 6.4 discusses solving trig equations and we want to explore some of the examples using Desmos to visualize the process in terms of graph intersections. We'll use that horizontal line technique quite a bit.

1. Read p.387 Example 1 and reproduce in Desmos the intersection of the sine function with the horizontal line at $y=\frac{1}{2}$. Verify that the intersection points correspond to the values in that example. You might want to change the horizontal axis to multiples-of-pi.

2. Read pp.388-89 Example 3 and reproduce the solution in Desmos as the intersection of a sinusoidal function and a horizontal line as shown on p.389. Use your Desmos graph to verify the solutions found in that example.

3. Now do the Try-it-Now on the bottom of p.389. First solve the problem using Desmos as you did in the previous two examples. Then, in your lab notebook, solve the problem algebraically following the technique of Example 3.

4. Using Desmos, reproduce a graphical presentation of the solutions to Example 7 on p.391 bottom. You'll need two equations (one of them will be $y=3$). Check the intersection points in the desired solution range and make sure they match the solutions in the example.

5. Using the technique of the previous step, do the Try-it-Now on p.391 in Desmos and check your answers.

6. Using Desmos, reproduce a graphical display of the solutions to Example 10 p.393. Also, read closely the analytical solution so you can learn to do these kinds of problems if you aren't able to use a graphical approach.

7. Do the Try-it-Now on p.393 using Desmos for a graphical solution and record your solutions in your lab notebook. Now solve the problem analytically using Example 10 p. 393 as a guide. Put your analytical solution in your lab notebook.