

Physics Lab 14: Pretty Periodic Patterns and more Musical Mathematics

Today's investigations are a follow-up to Physics Lab 13. You can still complete it even if you did not participate in that lab. In the last lab, you observed that pure frequencies of sound, such as those generated by a computer or on a tuning fork, could be modeled with a sinusoidal function. More interesting sounds, such as from pronouncing a vowel, whistling, or playing a musical instrument were also periodic, but much more complicated than a simple sinusoidal function. In today's lab, you will begin by working with a simulation that allows you to see the effect of combining simple pure sinusoidal functions of different frequencies to build up a complex periodic function. Next, you will listen to and then analyze musical notes.

Part I: Complex Periodic Functions from Simple Periodic Functions. estimated time: 45 minutes.

- Run the Fourier: Making Waves interactive simulation, available at <https://phet.colorado.edu/en/simulation/fourier>, then press the green Run Now! button and follow any prompts required to run the simulation.
- Make sure you are in the Discrete tab (this should be the default).
- Begin just by playing around (stay in the Discrete tab, please) and seeing what you can see and hearing what you can hear, getting a feel for the simulation. Take 5 – 10 minutes for this unstructured investigation. You may want to have your headphones in for part of this (note the Sound controls area); make sure you have decreased the default volumes in the program and on your computer to comfortable levels.
- Now, pursue a slightly more systematic investigation (as the sound is not necessarily pleasant, you can turn it off for this part). Do the following: Reset All; Set Graph controls to function of time (t); Turn on the Auto scale button. Investigate what happens if you increase each amplitude to 1 by itself (so all the other amplitudes are 0), and then to –1. What do you notice about the period of each successive function, compared to the previous?
- Now, investigate what happens to the Sum if you have more than one period at the same time: for example, set $A_1 = 1$ and $A_2 = 1$. Set $A_1 = 1$ and $A_2 = -1$. Set $A_1 = 1$ and $A_2 = 0.5$. Try $A_1 = -0.5$ and $A_3 = 1$. Add in more and more sin waves, etc.
- If you haven't already, try the different preset functions: triangle, square, sawtooth, wave packet.
- Does this give you a sense of how a complex periodic function can be built up of simple periodic functions?
- Now, go to the Wave Game tab. Play the Wave Game for a while. You switch levels using Game controls. It starts to become more challenging at Level 5, but the earlier levels give you good practice.

Part II: Listening to and Looking at Musical Notes. estimated time: 30 minutes.

- Make sure you have your headphones on, and that the volume on your computer is set to a comfortable level. If you don't have headphones, you can share with a neighbor or see if there are some available to borrow.
- Go to http://www.bgfl.org/bgfl/custom/resources_frp/client_frp/ks2/music/piano/. Note that what this program labels C, C#, D, D#, etc. is actually in the fourth octave, so should be called C4, C#4, D4, etc., and what this program labels C1 is actually C5. Don't worry too much about this.
- Make sure you are in piano mode. Spend some time playing the notes "in order": C, C#, D, D#, E, F, F#, G, G#, A, Bb, B, C1 (as in the note above, these actually should be labeled C4, C#4, D4, D#4, E4, F4, F#4, G4, G#4, A4, Bb4, B4, C5). What do you notice about the frequency of the notes as you progress "in order" (does the frequency decrease? increase? stay the same? jump around?)?
- Compare the piano notes to the flute notes. The easiest way to do this might be to have two tabs open in your web browser (or even two browser windows), one in piano mode and one in flute mode. Play C on the piano and on the flute. Play C# on the piano and on the flute. Especially pay attention to A and C1.
- Go to the program share, and under Handouts/Week 8 Lab, download Physics Lab 14 Notes (sorry, couldn't resist). You may print this out if you like, but this is not required. This file shows three graphs, one for each note. The first graph shows the Sound Pressure vs. Time. The second graph shows the FFT (Fast Fourier Transform) from 0 – 5000 Hz. If you weren't here for Physics Lab 13, ask one of your neighbors about the FFT graph. The third graph shows the FFT again, but this time zoomed in to focus on 200 – 500 Hz. Some terminology: the lowest frequency present is called the fundamental frequency. The higher frequencies are all integer multiples of the fundamental frequency, and are called harmonics (the fundamental is sometimes called the fundamental harmonic). The fundamental frequency is the pitch of the note you hear.
- Play each note while looking at its graphical representations. How do the third graphs in the series (which focus on the fundamental frequency for each note) represent the trend you hear?
- Consider your work in the Fourier: Making Waves simulation. Which graphs here correspond to which parts there?
- Now, play A on the piano and the flute while looking at the respective graphs (these are on the third page). Similarly, play C1 on the piano and the flute while looking at the respective graphs. While it is hard to tell since the FFT isn't zoomed in enough, the fundamental frequency for each note on each instrument is the same. In other words, the pitch of A on a piano and flute and the pitch of C1 on a piano and a flute are the same. However, what do you notice about

the higher harmonics? Consider both their presence and their relative strengths. It is the fundamental frequency which is the pitch of the note on any instrument, but it is the presence/absence and relative strength of the harmonics which gives each instrument its characteristic timbre (or tone quality), and that is a primary reason why the same note on different instruments sounds different.

- i) If you'd like to hear more notes, here's an online piano with more keys: <http://www.virtualpiano.net/>

Part III: Musical Mathematics. estimated time: 30 minutes.

- a) In the previous part, we noticed that the fundamental frequency increased as we played the notes "in order". Is there a pattern to the fundamental frequency of the successive notes? Is there a mathematical model to this pattern? We investigate that in this part of the lab.
- b) From the program fileshare, under Handouts/Week 8 Lab, download and save to your cubbie the LoggerPro file musicScale. This file has all the data used to produce the Physics Lab 14 Notes document you worked with in the previous part. You are welcome to use the data to look for all different kinds of patterns (all of the piano data sets are reasonably well named, and the flute data sets are acceptably named) on your own time. Here, you will focus in on a more directed investigation. For your convenience, the FFT graphs for each of the relevant notes are already produced, and zoomed in on the fundamental frequency for that note.
- c) Using the Examine tool, find the frequency of the fundamental note by looking for the frequency that corresponds to the middle of the curve in the zoomed in region shown. Example: for C4 piano, the fundamental frequency according to the data is 261.09 Hz. Copy a version of the table below into your lab notebook (you may want to use a vertically oriented table with three columns) and record the frequency in the appropriate spot.

"name"	C4	C#4	D4	D#4	E4	F4	F#4	G4	G#4	A4	Bb4	B4	C5
key number	40	41	42	43	44	45	46	47	48	49	50	51	52
fundamental frequency (Hz)													

- d) Find a reputable source for the frequency of piano notes. Cite the source, including why you found it reputable. How do your results match that source?
- e) In a new LoggerPro file, make a table with key numbers 40 - 52 and the associated fundamental frequency. Make a plot of fundamental frequency vs. key number. Recall that by conventions, this means the fundamental frequency is on the vertical axis and the key number on the horizontal axis. Just use the results from your data table, so your horizontal axis should run from 40 – 52.
- f) Zoom/change your axes scaling so that the horizontal axis runs from 0 up to 88 (the number of keys on a piano) and the vertical axis runs from 0 to 1000 Hz. There seems to be a pattern to this graph. Can we determine a mathematical model? So far, our mathematical models have included linear functions, quadratic functions, and sinusoidal functions (sin, tan, etc.). Let's see if any of these will do.
- g) Highlight the data, and fit a Linear Function. Copy this graph to a document for later printing. Does the linear fit look like it matches the data well? After having copied the graph for later printing, remove the linear fit.
- h) Repeat for a quadratic fit, making sure to copy the graph for later printing. Does the quadratic fit look like it matches the data well? While you can find a decent fit to the data using a quadratic, explain how you know that this can't be the correct model. After having copied the graph for later printing, remove the quadratic fit.
- i) Repeat for a sine fit, making sure to copy the graph for later printing. Does the sine fit look like it matches the data well? While you can find a decent fit to the data using a sine function, explain how you know that this can't be the correct model. After having copied the graph for later printing, remove the sine fit.
- j) So what is a better mathematical model? You'll soon see learn a new model function that is appropriate.
- k) Save this data file with an appropriate name to your cubbies.

If you finish early, complete Math Lab 8 if you have not already. Otherwise, work on this week's Problem Sets.