

## Lab 6: Measuring Momentum in 1 diMension

**Goals:** Improve communication, teamwork capacities, observational skills, and record-keeping; Explore momentum conservation in 1D.

**Equipment:** 2 meter track + feet + end stops + etc (should be set up); low friction carts: one blue and one red; masses: 2 silver or 1 black.

**Groups & Lab Notebook:** Groups of 2 for observations, followed by individual computer. Update Table of Contents. General Lab Notes guidelines.

### Sticking Collisions Observations:

In this series, two carts on low friction tracks will collide and stick together. These kinds of sticking collisions are also called *perfectly inelastic*. This will be achieved by having the Velcro ends of the carts oriented towards each other so that after any interaction, they stick together. Don't have any collisions be very violent – gentle collisions are best. You might find it hard to get the carts to move towards each other at the same speed when directed. This doesn't have to be exact, but they should move towards each other with similar speeds. Practice a few times.

**Scenario A: Equal masses, one cart stationary, sticking collision.** Place the red cart (mass  $m$ ) so that it is stationary on the track. Gently push and release the blue cart (mass  $m$ ) so that it moves to the right at speed  $v_i$  and collides with the stationary red cart. The blue cart and red cart should stick together and move together after the collision at some speed  $v_f$ . Draw before and after pictures for this scenario, including labels for the carts and indicating  $m$ ,  $v_i$ , and  $v_f$ . Draw the relative lengths of the velocity vectors to be qualitatively accurate (in other words, longer arrows mean moving faster, shorter arrows mean moving slower).

**Scenario B: Unequal masses, more massive cart stationary, sticking collision.** Add mass to the red cart (either two silver masses or one black mass) so that its mass is  $3m$ . Place the red cart with extra mass so that is stationary on the track. Gently push and release the blue cart (mass  $m$ ) so that it moves to the right at speed  $v_i$  and collides with the stationary red cart (note: this  $v_i$  should be about the same as before, but doesn't need to be exact). The blue cart and red cart should stick together and move together after the collision at some speed  $v_f$  (not necessarily the same as in the previous scenario). Draw before and after pictures for this scenario, including labels for the carts and indicating  $m$ ,  $3m$ ,  $v_i$ , and  $v_f$ . Draw the relative lengths of the velocity vectors to be qualitatively accurate.

**Scenario C: Unequal masses, more massive cart moving, sticking collision.** Add mass to the blue cart so that its mass is  $3m$ . Place the red cart (mass  $m$ ) so that is stationary on the track. Gently push and release the blue cart with extra mass so that it moves to the right at speed  $v_i$  and collides with the stationary red cart (note: this  $v_i$  should be about the same as before, but doesn't need to be exact). The blue cart and red cart should stick together and move together after the collision at speed  $v_f$  (not necessarily the same as the previous scenario). Draw before and after pictures for this scenario, including labels for the carts and indicating  $m$ ,  $3m$ ,  $v_i$ , and  $v_f$ . Draw the relative lengths of the velocity vectors to be qualitatively accurate.

**Scenario D: Unequal masses, both carts moving about same speed, sticking collision.** Add mass to the blue cart so that its mass is  $3m$ . Gently push and release the blue cart (mass  $3m$ ) so that it moves to the right at speed  $v_i$  and collides with the red cart (mass  $m$ ) moving to the left at about the same speed  $v_i$ . The blue cart and red cart should stick together and move together after the collision at some speed  $v_f$ . Draw before and after pictures for this scenario, including labels for the carts and indicating  $m$ ,  $3m$ ,  $v_i$ ,  $v_i$ , and  $v_f$ . Draw the relative lengths of the velocity vectors to be qualitatively accurate.

**Scenario E: Equal masses, both carts moving at different speeds, sticking collision.** Gently push and release the blue cart (mass  $m$ ) so that it moves to the right at speed  $v_{B,i}$  and collides with the red cart (mass  $m$ ) moving to the left at speed  $v_{R,i}$ ; have the blue cart move faster than the red cart. The blue cart and red cart should stick together and move together after the collision at some speed  $v_f$ . Draw before and after pictures for this scenario, including labels for the carts and indicating  $m$ ,  $v_{B,i}$ ,  $v_{R,i}$ , and  $v_f$ . Draw the relative lengths of the velocity vectors to be qualitatively accurate.

**Scenario F: Unequal masses, both carts moving with massive car moving faster, sticking collision.** Add mass to the red cart so that its mass is  $3m$ . Gently push and release the blue cart (mass  $m$ ) so that it moves to the right at speed  $v_{B,i}$  and collides with the red cart (mass  $3m$ ) moving to the left at speed  $v_{R,i}$ ; have the red cart move faster than the blue cart. The blue cart and red cart should stick together and move together after the collision at some speed  $v_f$ . Draw before and after pictures for this scenario, including labels for the carts and indicating  $m$ ,  $3m$ ,  $v_{B,i}$ ,  $v_{R,i}$ , and  $v_f$ . Draw the relative lengths of the velocity vectors to be qualitatively accurate.

### **Bouncing Collisions Observations:**

In this series, the carts will collide and bounce off each other. This will be achieved by having magnet ends oriented towards each other so that after the collision they don't stick together.

**Scenario G: Equal masses, one cart stationary, bouncing collision.** Place the red cart (mass  $m$ ) so that is stationary on the track. Gently push and release the blue cart (mass  $m$ ) so that it moves to the right at speed  $v_i$  and collides with the stationary red cart (mass  $m$ ). The blue cart and red cart should bounce and move separately after the collision, with the blue cart moving at  $v_{B,f}$  (though you might note something interesting about  $v_{B,f}$ ) and the red cart moving at  $v_{R,f}$ . Draw before and after pictures for this scenario, including labels for the carts and indicating  $m$ ,  $v_i$ ,  $v_{B,f}$ , and  $v_{R,f}$ . Draw the relative lengths of the velocity vectors to be qualitatively accurate.

**Scenario H: Unequal masses, more massive cart stationary, bouncing collision.** Add mass to the red cart so that its mass is  $3m$ . Place the red cart with extra mass so that is stationary on the track. Gently push and release the blue cart (mass  $m$ ) so that it moves to the right at speed  $v_i$  and collides with the stationary red cart (mass  $3m$ ). The blue cart and red cart should bounce and move separately after the collision, with the blue cart moving at  $v_{B,f}$  (though you might note something interesting about  $v_{B,f}$ ) and the red cart moving at  $v_{R,f}$ . Draw before and after pictures for this scenario, including labels for the carts and indicating  $m$ ,  $3m$ ,  $v_i$ ,  $v_{B,f}$ , and  $v_{R,f}$ . Draw the relative lengths of the velocity vectors to be qualitatively accurate.

**Scenario I: Unequal masses, more massive cart moving, bouncing collision.** Add mass to the blue cart so that its mass is  $3m$ . Place the red cart (mass  $m$ ) so that is stationary on the track. Gently push and release the blue cart (mass  $3m$ ) so that it moves to the right at speed  $v_i$  and collides with the stationary red cart (mass  $m$ ). The blue cart and red cart should bounce and move separately after the collision, with the blue cart moving at  $v_{B,f}$  and the red cart moving at  $v_{R,f}$ . Draw before and after pictures for this scenario, including labels for the carts and indicating  $m$ ,  $3m$ ,  $v_i$ ,  $v_{B,f}$ , and  $v_{R,f}$ . Draw the relative lengths of the velocity vectors to be qualitatively accurate.

**Scenario J: Equal masses, both carts moving at about same speed, bouncing collision.** Gently push and release the blue cart (mass  $m$ ) so that it moves to the right at speed  $v_i$  and collides with the red cart (mass  $m$ ) moving to the left at about the same speed  $v_i$ . The blue cart and red cart should bounce and move separately after the collision, with the blue cart moving at  $v_{B,f}$  and the red cart moving at  $v_{R,f}$ . Draw before and after pictures for this scenario, including labels for the carts and indicating  $m$ ,  $v_i$ ,  $v_{B,f}$ , and  $v_{R,f}$ . Draw the relative lengths of the velocity vectors to be qualitatively accurate.

**Scenario K: Unequal masses, both carts moving at about same speed, bouncing collision.** Add mass to the red cart so that its mass is  $3m$ . Gently push and release the blue cart (mass  $m$ ) so that it moves to the right at speed  $v_i$  and collides with the red cart (mass  $3m$ ) moving to the left at about the same speed  $v_i$ . The blue cart and red cart should bounce and move separately after the collision, with the blue cart moving at  $v_{B,f}$  and the red cart moving at  $v_{R,f}$  (though you might note something interesting about  $v_{R,f}$ ). Draw before and after pictures for this scenario, including labels for the carts and indicating  $m$ ,  $v_i$ ,  $v_{B,f}$ , and  $v_{R,f}$ . Draw the relative lengths of the velocity vectors to be qualitatively accurate.

### **Mystery Scenarios:**

**Mystery Scenario I:** Consider a collision between a light cart and a massive cart that are both moving (relative to the track). Can you have a collision such that the massive cart comes to rest after the collision? Why or why not?

**Mystery Scenario II:** Consider a collision between two carts of similar mass that are moving towards each other at about the same speed. Can you have a collision such that the carts move away from each other faster than they were moving towards each other? Why or why not?

**After discussing the Mystery Scenarios in your group, call me over for discussion and demonstration.**

### **Analysis: (work at your own computer but you're welcome to work near and consult with your lab partner)**

- Consider Scenario A. Explain why you can treat the momentum of the carts as conserved. Use your before and after diagrams and conservation of momentum to show that  $v_f = \frac{1}{2} v_i$ .
- Consider Scenario B. Use your before and after diagrams and conservation of momentum to show that  $v_f = \frac{1}{4} v_i$ .
- Consider Scenario C. Use your before and after diagrams and conservation of momentum to show that  $v_f = \frac{3}{4} v_i$ .

- d) Consider Scenario D. Use your before and after diagrams and conservation of momentum to show that  $v_f = \frac{1}{2} v_i$ .
- e) Consider Scenario E. Use your before and after diagrams and conservation of momentum to explain why the stuck-together carts move to the right after the collision.
- f) Consider Scenario F. Use your before and after diagrams and conservation of momentum to explain why the stuck-together carts move to the left after the collision.
- g) Consider Scenarios G – K. Why is a momentum analysis more complicated in these scenarios (compared to the sticking collisions?) Are you able to determine the final velocities of each cart in terms of the initial velocities (and cart mass) like you could in Scenarios A – D? Why or why not?

**Discuss your analysis with me.**

**Further Analysis:**

In the program share, Handouts: Lab 6, you will find a series of LoggerPro files that correspond to the scenarios you observed. Copy the files to a usefully named folder in your Cubbie. The data was collected using two motion detectors, one aimed at the red cart (Position 1 data, in red) and one aimed at the blue cart (Position 2 data, in blue). Each file shows the position vs. time graph for the motion of the carts before, during, and after the collision described in the corresponding scenario.

- 1) How do you determine velocity from a position vs. time graph? How do you get that information in LoggerPro?
- 2) Choose a Sticking Collision scenario you found interesting or surprising or that would give you the best practice with momentum conservation problem solving. Examine the graph, and identify regions just before the collision, the collision event, and just after the collision. Adjust the graph to show individual points. Highlight 3 or 4 points just before the collision, and determine the velocity of each cart (keep track of signs). Highlight 3 or 4 points just after the collision and determine the velocity of the (combined) carts. Make sure not to include the actual collision time.
- 3) Determine the momentum of each individual cart before and the combined cart after the collision. Note that  $m = 250$  g (approximately). Make sure to keep track of the sign of each momentum.
- 4) Compare the total momentum before the collision to the total momentum after the collision. Note that the low friction carts are not friction-less. Given that, are your results broadly consistent with conservation of momentum?
- 5) Choose a Bouncing Collision Scenario you found interesting or surprising, and repeat the above analysis. Is your analysis broadly consistent with conservation of momentum?
- 6) We (should have) discussed earlier in this investigation that for bouncing collisions, momentum conservation was insufficient to determine the final velocity of each cart in terms of the initial velocities (and cart masses). However, the bouncing collisions observed in today's investigation can be treated as nearly *perfectly elastic*, since the magnets act as nearly perfect springs. Using your results from the Bouncing Collision Scenario you just analyzed, do you find that your results are broadly consistent with the claim that this collision is nearly perfectly elastic?
- 7) Perfectly elastic collisions conserve momentum (a vector quantity) and kinetic energy (a scalar quantity). As you have or will see, solving problems involving elastic collisions is sometimes algebraically challenging. A nice result for 1D elastic collisions involving 2 objects (so a very special case, but a useful one) combines the equations that come from conserving these quantities into a relationship that you can use alongside momentum conservation. If you have an object of mass  $m_1$  moving with velocity  $v_{1i}$  (note that this can be positive or negative depending on direction) and an object of mass  $m_2$  moving with velocity  $v_{2i}$  (again, can be positive or negative) before the perfectly elastic 1D collision, and after the collision,  $m_1$  moves with  $v_{1f}$  and  $m_2$  with  $v_{2f}$ , we can relate  $v_{1i}$ ,  $v_{2i}$ ,  $v_{1f}$  and  $v_{2f}$  as follows:

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

(note the order and the negative signs. Recall that any of the input quantities can also be positive or negative depending on the situation)

Is the results of your analysis of the Bouncing Collisions Scenario (broadly) consistent with this result?