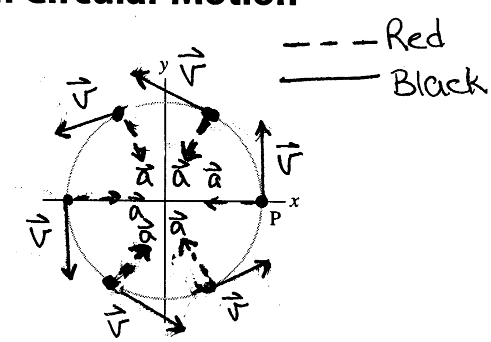
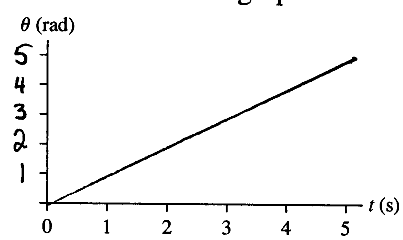
6 Circular Motion, Orbits, and Gravity

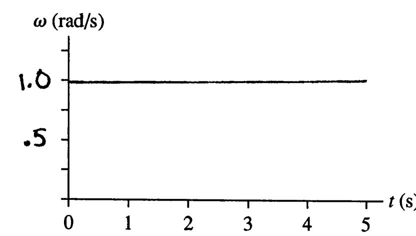
6.1 Uniform Circular Motion

6.2 Speed, Velocity, and Acceleration in Uniform Circular Motion

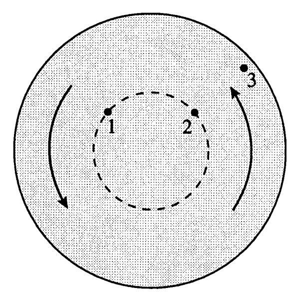
1. A particle undergoes uniform circular motion with constant angular velocity $\omega = +1.0$ rad/s, starting from point P.
   a. On the figure, draw a motion diagram showing the location of the particle every 1.0 s until the particle has moved through an angle of 5 rad. Draw velocity vectors **black** and acceleration vectors **red**. For this question, you can use 1 rad $\approx 60°$.
   b. Below, graph the particle's angular position θ and angular velocity ω for the first 5 s of motion. Include an appropriate vertical scale on both graphs.







2. The figure shows three points on a steadily rotating wheel.
   a. Rank in order, from largest to smallest, the angular velocities ω_1, ω_2, and ω_3 of these points.



Order: $\omega_1 = \omega_2 = \omega_3$

Explanation: Each point traverses the same angle in the same time. All points on the wheel rotate with the same period.

   b. Rank in order, from largest to smallest, the speeds v_1, v_2, and v_3 of these points.

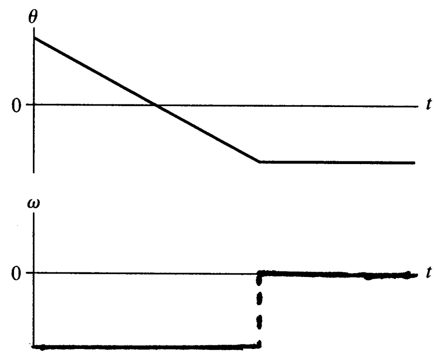
Order: $v_3 > v_1 = v_2$

Explanation:

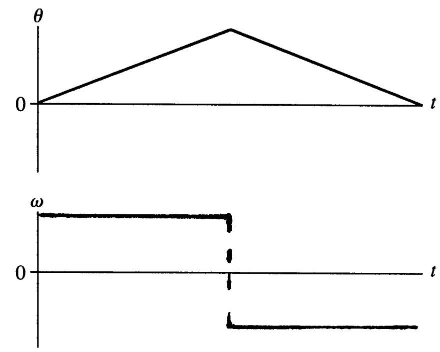
$v = \omega r$, so the points at larger r are moving faster.

3. Below are two angular position-versus-time graphs. For each, draw the corresponding angular velocity-versus-time graph directly below it.

a.

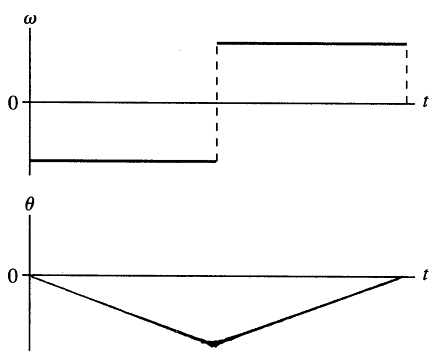


b.



4. Below are two angular velocity-versus-time graphs. For each, draw the corresponding angular position-versus-time graph directly below it. Assume $\theta_0 = 0$ rad.

a.



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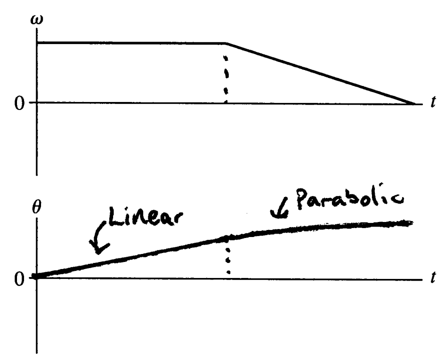
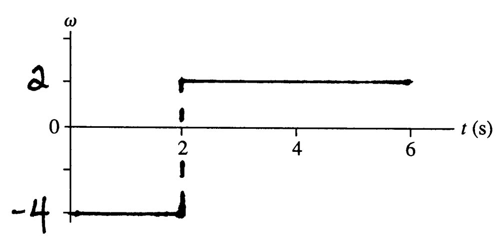
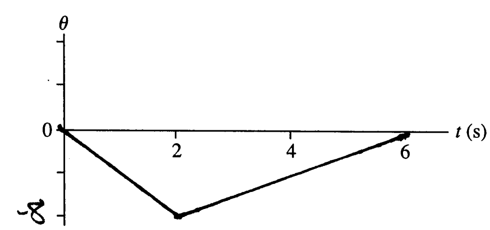


5. A particle in circular motion rotates clockwise at 4 rad/s for 2 s, and then counterclockwise at 2 rad/s for 4 s. The time required to change direction is negligible. Graph the angular velocity and the angular position, assuming $\theta_0 = 0$ rad.





6. A particle moves in uniform circular motion with $a = 8$ m/s^2. What is a if

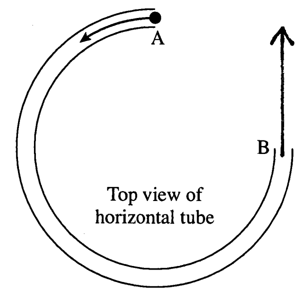
a. The radius is doubled without changing the angular velocity? $a_r = 16\ \frac{m}{s^2}$

b. The radius is doubled without changing the particle's speed? $a_r = 4\ \frac{m}{s^2}$

c. The angular velocity is doubled without changing the circle's radius? $a_r = 32\ \frac{m}{s^2}$

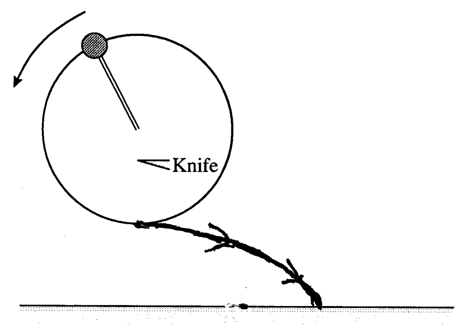
6.3 Dynamics of Uniform Circular Motion

7. The figure shows a *top view* of a plastic tube that is fixed on a horizontal table top. A marble is shot into the tube at A. Sketch the marble's trajectory after it leaves the tube at B. Explain.



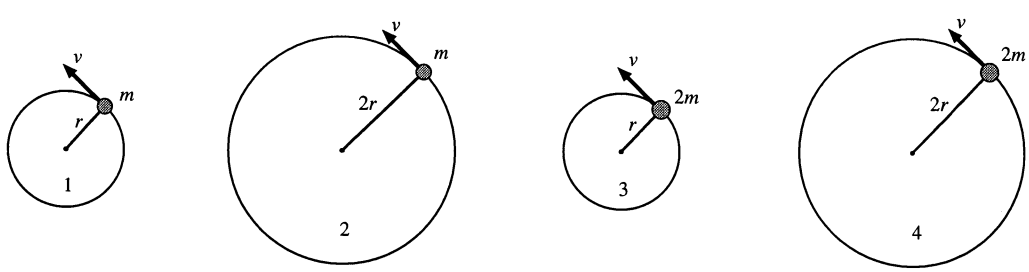
The marble continues in a straight line (towards the top of the page).

8. A ball swings in a *vertical* circle on a string. During one revolution, a very sharp knife is used to cut the string at the instant when the ball is at its lowest point. Sketch the subsequent trajectory of the ball until it hits the ground. Explain.



The trajectory is parabolic, like that of a horizontally launched projectile.

9. The figures are a bird's-eye view of particles moving in horizontal circles on a table top. All are moving at the same speed. Rank in order, from largest to smallest, the tensions T_1 to T_4.

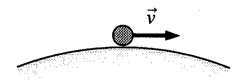


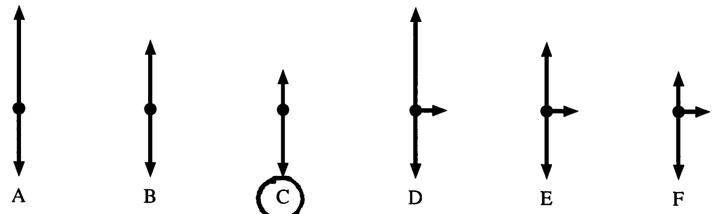
Order: $T_3 > T_1 = T_4 > T_2$

Explanation:

$T = \frac{mv^2}{r}$ Case 3 combines larger mass and smaller r. Case 4 is the same as case 1 because both the mass and the radius are doubled.

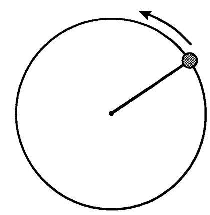
10. A ball rolls over the top of a circular hill. Rolling friction is negligible. Circle the letter of the ball's free-body diagram at the very top of the hill.



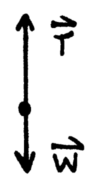


Explanation: At that instant, only the direction of motion is changing so the weight is greater than the normal force and the ball is accelerating downward (toward the center of the circular hill).

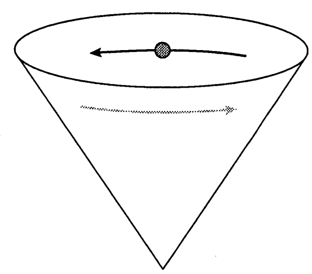
11. A ball on a string moves in a vertical circle. When the ball is at its lowest point, is the tension in the string greater than, less than, or equal to the ball's weight? Explain. (You should include a free-body diagram as part of your explanation.)

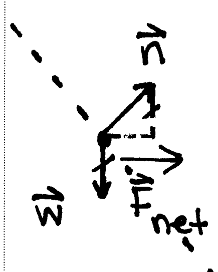


At the lowest point, the acceleration is upward. Thus, the tension must be greater than the weight for the net force to be upward.



12. A marble rolls around the inside of a cone. Draw a free-body diagram of the marble when it is on the left side of the cone, coming toward you.



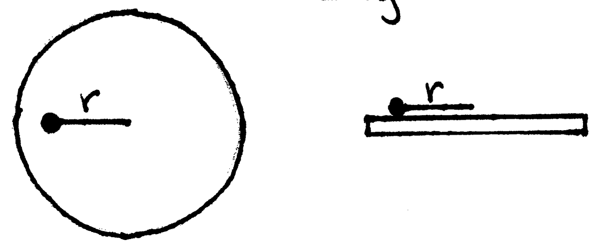


13. A coin of mass m is placed distance r from the center of a turntable. The coefficient of static friction between the coin and the turntable is μ_s. Starting from rest, the turntable is gradually rotated faster and faster. At what angular velocity does the coin slip and "fly off"?

PSS 6.1

a. Begin with a visual overview. Draw the turntable both as seen from above and as an edge view with the coin on the left side coming toward you. Label radius r, make a table of known information, and indicate what you're trying to find.

Above Edge



Known: m, r, μ_s, $v_0 = 0$

Find: ω

b. What direction does $\vec{f}_s$ point? towards the center of the turntable

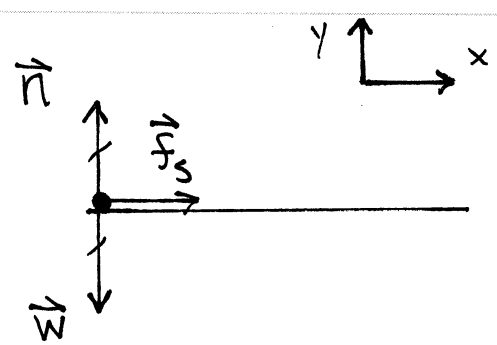
Explain.

$\vec{f}_s$ is the force causing the coin to undergo circular motion.

c. What condition describes the situation just as the coin starts to slip? Write this condition as a mathematical statement.

$$f_{s_{max}} = ma$$

d. Now draw a free-body diagram of the coin. Following Problem Solving Strategy 6.1, draw the free-body diagram with the circle viewed edge on, the x-axis pointing toward the center of the circle, and the y-axis perpendicular to the plane of the circle. Your free-body diagram should have three forces on it.



e. Referring to Problem Solving Strategy 6.1, write Newton's second law for the x- and y-components of the forces. One sum should equal 0, the other mv^2/r.

$$\Sigma F_x = f_s = \frac{mv^2}{r}$$

$$\Sigma F_y = n - mg = 0$$

f. The two equations of part e are valid for any angular velocity up to the point of slipping. If you combine these with your statement of part c, you can solve for the speed v_{max} at which the coin slips. Do so.

$$f_{s_{max}} = \mu_s n = \mu_s mg = \frac{m v_{max}^2}{r}$$

$$\text{or} \quad v_{max} = \sqrt{\mu_s r g}$$

g. Finally, use the relationship between v and ω to find the angular velocity of slipping.

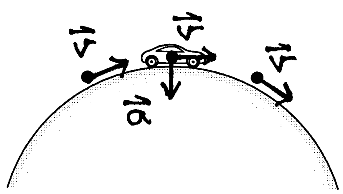
$$\omega = \frac{v}{r} = \sqrt{\frac{\mu_s g}{r}}$$

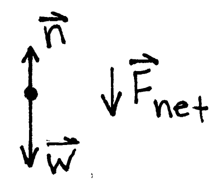
6.4 Apparent Forces in Circular Motion

14. The drawing is a partial motion diagram for a car rolling at constant speed over the top of a circular hill.

    a. Complete the motion diagram by adding the car's velocity vectors. Then use the velocity vectors to determine *and show* the car's acceleration $\vec{a}$ at the top of the hill.

    b. To the right, draw a free-body diagram of the car at the top of the hill. Next to the free-body diagram, indicate the direction of the net force on the car or, if appropriate, write $\vec{F}_{net} = \vec{0}$.

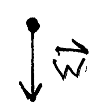




    c. Does the net force point of your free-body diagram point in the same direction you showed for the car's acceleration? yes, downward
    If not, you may want to reconsider your work thus far because Newton's second law requires $\vec{F}_{net}$ and $\vec{a}$ to point the same way.

    d. Is there a maximum speed at which the car can travel over the top of the hill and not lose contact with the ground? If so, show how the free-body diagram would look at that speed. If not, why not?

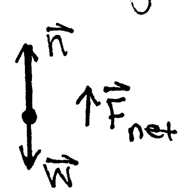
    yes, if $F_{net} = w$



15. A stunt plane does a series of vertical loop-the-loops. At what point in the circle does the pilot feel the heaviest? Explain. Include a free-body diagram with your explanation.

The pilot feels heaviest at the bottom of the vertical loop. At that point, the normal force on the pilot is greatest, as is the apparent weight.

At bottom:



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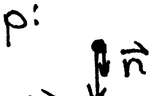


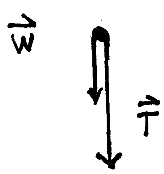
This analysis assumes the pilot is moving at comparable speed throughout the loop.

16. A roller-coaster car goes around the inside of a loop-the-loop. One of the following statements is true at the highest point in the loop, and one is true at the lowest point. Check the true statements.

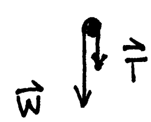
	Highest	Lowest
The car's apparent weight w_{app} is always less than w		
The car's apparent weight w_{app} is always equal to w		
The car's apparent weight w_{app} is always greater than w		✓
w_{app} could be less than, equal to, or greater than w	✓	

17. You can swing a ball on a string in a *vertical* circle if you swing it fast enough.

a. Draw two free-body diagrams of the ball at the top of the circle. On the left, show the ball when it is going around the circle very fast. On the right, show the ball as it goes around the circle more slowly.



Very fast



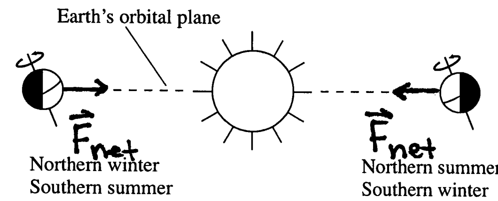
Slower

b. Suppose the ball has the smallest possible frequency that allows it to go all the way around the circle. What is the tension in the string when the ball is at the highest point? Explain.

$\vec{T} = 0$. At the smallest frequency, the only radially inward force is the force of gravity, the weight.

6.5 Circular Orbits and Weightlessness

18. The earth has seasons because the axis of the earth's rotation is tilted 23° away from a line perpendicular to the plane of the earth's orbit. You can see this in the figure, which shows the edge of the earth's orbit around the sun. For both positions of the earth, draw a force vector to show the net force acting on the earth or, if appropriate, write $\vec{F} = \vec{0}$.



19. A small projectile is launched parallel to the ground at height $h = 1$ m with sufficient speed to orbit a completely smooth, airless planet. A bug rides in a small hole inside the projectile. Is the bug weightless? Explain.

The bug is weightless in the sense that it is in freefall with the projectile. The bug still has a weight of $\vec{w} = m_{bug} g$.

20. It's been proposed that future space stations create "artificial gravity" by rotating around an axis. (The space station would have to be much larger than the present space station for this to be feasible.)

a. How would this work? Explain.

The rotational motion would require a force towards the center of the circle, which would be supplied by the floor of the space station. The "artificial gravity" would arise from the experience of apparent weight from the normal force preventing the objects and occupants from flying off tangentially.

b. Would the artificial gravity be equally effective throughout the space station? If not, where in the space station would the residents want to live and work?

As in a centrifuge, the artificial gravity is strongest at the greatest radius. The "floor" would be on the inside of the outermost surface from the axis of rotation.

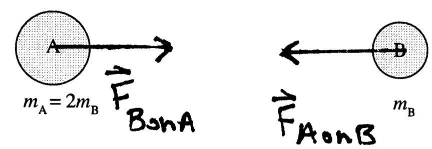
6.6 Newton's Law of Gravity

21. Is the earth's gravitational force on the sun larger than, smaller than, or equal to the sun's gravitational force on the earth? Explain.

The gravitational forces are equal and opposite. They are an action-reaction pair.

22. Star A is twice as massive as star B.

a. Draw gravitational force vectors on both stars. The length of each vector should be proportional to the size of the force.



b. Is the acceleration of star A toward B larger than, smaller than, or equal to the acceleration of star B toward A? Explain.

The acceleration of star A is smaller. For equal forces, the smaller mass experiences the greater acceleration because $a = F/m$.

23. The quantity y is inversely proportional to the square of x, and $y = 4$ when $x = 5$.

a. Write an equation to represent this inverse-square relationship for all y and x.

$y = \frac{100}{x^2}$ $(k = yx^2 = 100)$

b. Find y if $x = 2$. $y = 25$

c. Find x if $y = 100$. $x = 1$

24. The quantity y is inversely proportional to the square of x. For one value of x, $y = 12$.

a. What is the value of y if x is doubled? $y = 3$

b. What is the value of y if the original value of x is halved? $y = 48$

$y = \frac{c}{x^2}$ a. $12 = \frac{c}{x^2} \Rightarrow \frac{12}{4} = \frac{c}{(2x)^2}$

b. $\frac{12}{1/4} = \frac{c}{(x/2)^2}$

25. How far away from the earth does an orbiting spacecraft have to be in order for the astronauts inside to be "weightless?"

The astronauts can be "weightless" at any distance because an object is said to be weightless if it is in freefall (as in orbit). For the gravitational force to become zero, the spacecraft would have to be an infinite distance away.

26. The acceleration due to gravity at the surface of Planet X is 20 m/s^2. The radius and the mass of Planet Z are twice those of Planet X. What is g on Planet Z?

$g \propto \frac{m}{r^2}$ $\quad g_2 = g_1 \frac{2}{(2)^2} = \frac{g_1}{2} = \boxed{10 \frac{m}{s^2}}$

6.7 Gravity and Orbits

27. Planet X orbits the star Omega with a "year" that is 200 earth days long. Planet Y circles Omega at four times the distance of Planet X. How long is a year on Planet Y?

From Kepler's third law, the orbital period squared is proportional to the orbital radius cubed. $T^2 \propto r^3$.

Thus, at $r_Y = 4r_x$, $T_Y^2 \propto (4r_x)^3 = 64 r_x^3 \propto (8T_x)^2$

$T_Y = 8T_x$. A year on planet Y is 1600 earth days long.

28. The mass of Jupiter is $M_{Jupiter} = 300M_{earth}$. Jupiter orbits around the sun with $T_{Jupiter} = 11.9$ years in an orbit with $r_{Jupiter} = 5.2r_{earth}$. Suppose the earth could be moved to the distance of Jupiter and placed in a circular orbit around the sun. The new period of the earth's orbit would be

a. 1 year.
b. 11.9 years.
c. Between 1 year and 11.9 years.
d. More than 11.9 years.
e. It could be anything, depending on the speed the earth is given.
f. It is impossible for a planet of earth's mass to orbit at the distance of Jupiter.

Circle the letter of the true statement. Then explain your choice.

The orbital period is independent of the mass of the orbiting body, provided that the orbiting body's mass is much less than the mass of the body being orbited.

29. The gravitational force of a star on orbiting planet 1 is F_1. Planet 2, which is twice as massive as Planet 1 and orbits at twice the distance from the star, experiences gravitational force F_2.

a. What is the ratio F_2/F_1?

$$F_1 = \frac{-GMm_1}{r_1^2} \quad \text{so } F_2 = \frac{-GM(2m_1)}{(2r_1)^2} \qquad (M = \text{mass of star})$$

$$\frac{F_2}{F_1} = \frac{\frac{-GM(2m_1)}{(2r_1)^2}}{\frac{-GMm_1}{r_1^2}} = \frac{2}{2^2} = \boxed{\frac{1}{2}}$$

b. Planet 1 orbits the star with period T_1 and Planet 2 with period T_1. What is the ratio T_2/T_1?

$$T_1^2 = \left(\frac{4\pi^2}{GM}\right) r_1^3 \quad \text{so } T_2^2 = \left(\frac{4\pi^2}{GM}\right)(2r_1)^3 = 8T_1^2$$

$$\frac{T_2}{T_1} = \left(\frac{T_2^2}{T_1^2}\right)^{1/2} = \left(\frac{8}{1}\right)^{1/2} = 2\sqrt{2} = \boxed{2.83}$$

30. Satellite A orbits a planet with a speed of 10,000 m/s. Satellite B, orbiting at the same distance from the center of the planet, is twice as massive as Satellite A. What is the speed of Satellite B?

10,000 m/s The speeds are the same because the mass of the satellite is not relevant here. Only the radius of the orbit and mass of the planet are needed to find the speed.

$$\frac{GM\cancel{m}}{r^2} = \frac{\cancel{m}v^2}{r} \qquad v = \sqrt{\frac{GM_{planet}}{r_{orbit}}}$$